Homework 8

P7.2.15 The voltage waveform of Figure P7.2.15 is applied to an initially-uncharged 0.5 μH inductor. (a) Derive expressions for the inductor current during the time intervals:

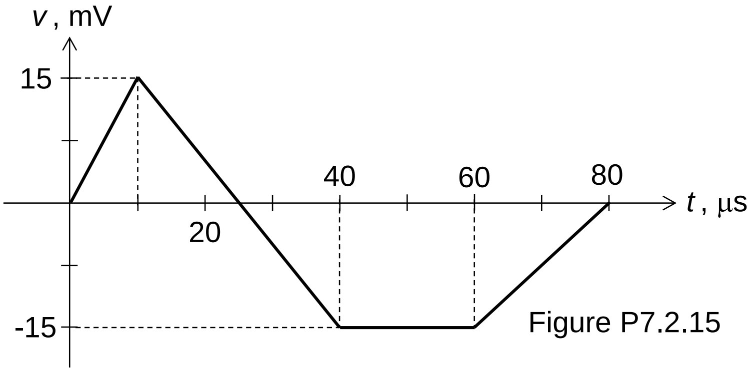


Figure P7.2.15

$0 \leq t \leq 10$ μs, $10 \leq t \leq 40$ μs, $40 \leq t \leq 60$ μs, $60 \leq t \leq 80$ μs, and $t > 80$ μs. (b) What is the flux linkage in the inductor at $t = 10$ μs and at $t = 50$ μs? Check the final value of current against the final flux linkage. (c) What is the energy stored in the inductor at $t = 80$ μs? (d) How do the expressions for the current through the inductor derived in (a) above change it the inductor current was initially 0.5 A?

Solution: $i(t) = \frac{1}{L}\int_{t_0}^{t} v dt + I(t_0)$ (a) $0 \leq t \leq 10\,\mu$s: $i(t) = \frac{1}{0.5}\int_0^t 1.5tdt = 1.5t^2$ mV, where t is in μs. At t = 10 μs, $i(t)$ = 150 mA.

$10 \leq t \leq 40\,\mu$s: $i(t) = \frac{1}{0.5}\int_{10}^{t}(-t+25)dt + 150 = \left[-t^2 + 50t\right]_{10}^{t} + 150$

$= -t^2 + 50t - 250$ mA. At t = 40 μs, $i(t)$ = 150 mA.

$40 \leq t \leq 60\,\mu$s: $i(t) = \frac{1}{0.5}\int_{40}^{t} -15dt + 150 = \left[-30t\right]_{40}^{t}$ +150 = –30t + 1350 mA.

At t = 60 μs, $i(t)$ = –450 mA.

$60 \leq t \leq 80\,\mu$s: $i(t) = \frac{1}{0.5}\int_{60}^{t}(0.75t - 60)dt - 450 = \left[0.75t^2 - 120t\right]_{60}^{t} - 450 =$

$0.75t^2 - 120t + 4050$ mA. At t = 80 μs, $i(t)$ = –750 mA.

$t \geq 80\,\mu$s: $i(t)$ = –750 mA.

Check: Total area = 0.5×15×10 + 0.5×15×15 – 0.5×15×15 – 15×20 – 0.5×15×20 = –375 nWb-T. Hence $i(t)$ = $\frac{-375}{0.5} = -750$ mA.

(b) At $t = 10\,\mu$s, $i(t) = 150$ mA, so $\lambda = Li = 75$ nWb-T.

At $t = 50\,\mu$s, $i(t) = -150$ mA, so $\lambda = Li = -75$ nWb-T.

(c) At $t = 80\,\mu$s, $i(t)$ = –750 mA, so $w = \frac{1}{2}Li^2 = 0.14\,\mu$J.

(d) All the expressions derived above for the current are increased by 0.5 A.

P7.2.21 Determine the energy stored in the inductor in Figure P7.2.21, assuming a dc steady state.

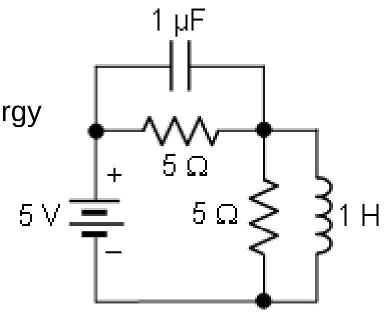


Figure P7.2.21

Solution: The current through the inductor is 5/5 = 1 A. The energy stored in the inductor is (1/2)×1×1 = 0.5 J.

P7.3.6 Determine the equivalent inductance between terminals ‘ab’ in Figure 7.3.6, assuming all inductances are 0.5 H.

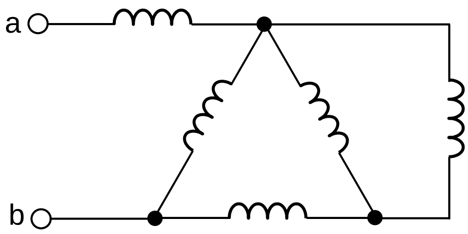


Figure P7.3.6

Solution: 0.5 H in parallel with 0.5 H is 0.25 H; in series with 0.5 H, this gives 0.75 H. In parallel with 0.5 H, this gives 0.3 H. It follows that L_{eq} = 0.3 + 0.5 = 0.8 H.

P7.4.4 Given the circuit of Figure P7.4.4. (a) Derive the dual circuit; (b) compare the resistance or conductance seen by the independent source in each circuit; (c) compare the power delivered or absorbed by each circuit element in the two circuits.

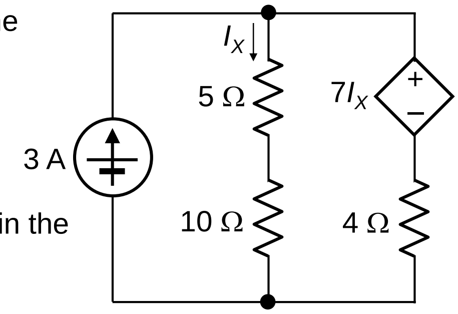


Figure P7.4.4

Solution: (a) Bearing in mind that the dual of a series connection is a parallel connection, the dual circuits are as shown, with each element in one circuit being the dual of the corresponding element in the other circuit. Note that in (a) the polarity of the CCVS is a voltage drop when the mesh is traversed CW. In (b) the current of the VCCS leaves the node that is the dual of the mesh in (a).

(b) It is seen from the currents and voltages that the CCVS in (a) is equivalent to a resistance of 7/2 = 3.5 Ω, so that the resistance seen by the source is 15||7.5 = 5 Ω. Similarly, the VCCS in (b) is equivalent to a conductance of 7/2 = 3.5 S, so that the conductance seen by the source is 15×7.5/22.5 = 5 S.

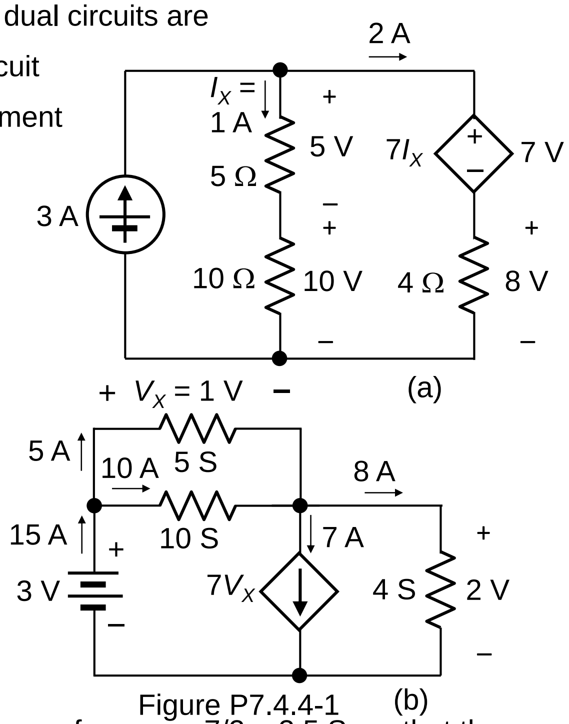
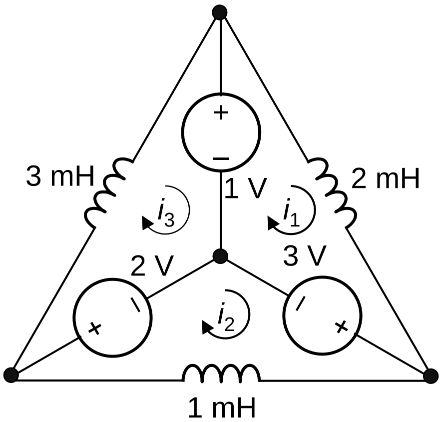


Figure P7.4.4-1

(c) It is seen that the voltages and currents are interchanged for dual elements, so that the powers are the same. These powers are: independent sources deliver 45 W, 5 Ω/5 S resistors dissipate 5 W, 10 Ω/10 S resistors dissipate 10 W, 4 Ω/4 S resistors in parallel with dependent source dissipate 16 W, dependent sources absorb 14 W.

P7.4.6 (a) Derive the mesh-current equations using the mesh currents shown in Figure P7.46; (b) deduce the node-voltage equations of the dual circuit; (c) derive the circuit that will give these node-voltage equations with respect to a specified reference node.



(a)

Solution: (a) The mesh-current equations are:

$j\omega 2\times10^{-3} i_1 = 1\text{ V} - 3\text{ V}$

$j\omega 1\times10^{-3} i_2 = 3\text{ V} - 2\text{ V}$

$j\omega 3\times10^{-3} i_3 = 2\text{ V} - 1\text{ V}$

(b) The node-voltage equations of the dual circuit are:

$j\omega 2\times10^{-3} v_1 = 1\text{ A} - 3\text{ A}$

$j\omega 1\times10^{-3} v_2 = 3\text{ A} - 2\text{ A}$

$j\omega 3\times10^{-3} v_3 = 2\text{ A} - 1\text{ A}$

(c) The circuit that gives these node-voltage equations is as shown.

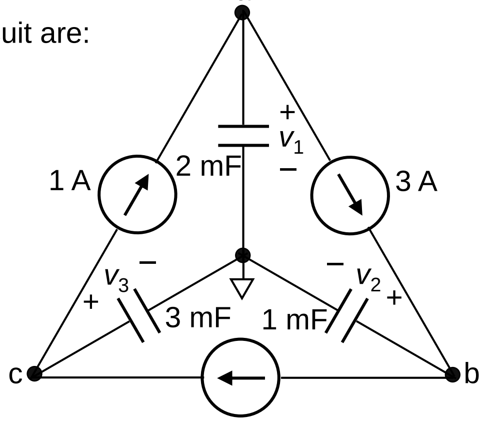


Figure P7.4.6-1 (b)